



**Banha University**  
**Faculty of Engineering - Shoubra**  
**Civil Engineering Department**

**REINFORCED CONCRETE 1 - B**

**For 2<sup>nd</sup> Year Civil – 2<sup>nd</sup> Term**

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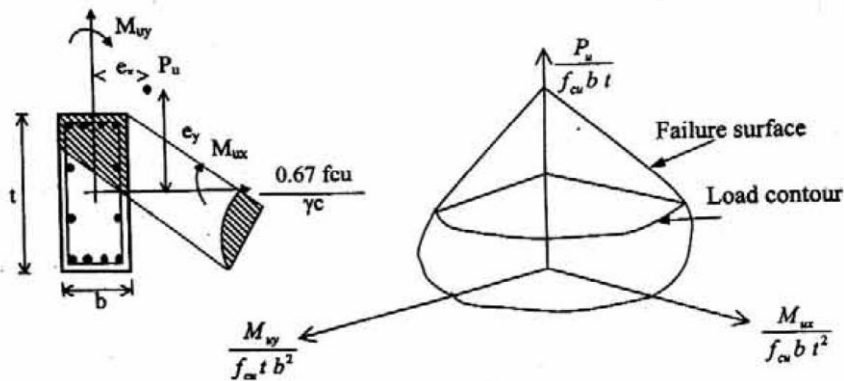
**Chapter (4)**

**Column Sections Under Biaxial Bending**

# Chapter 4 Column Sections Under Biaxial Bending

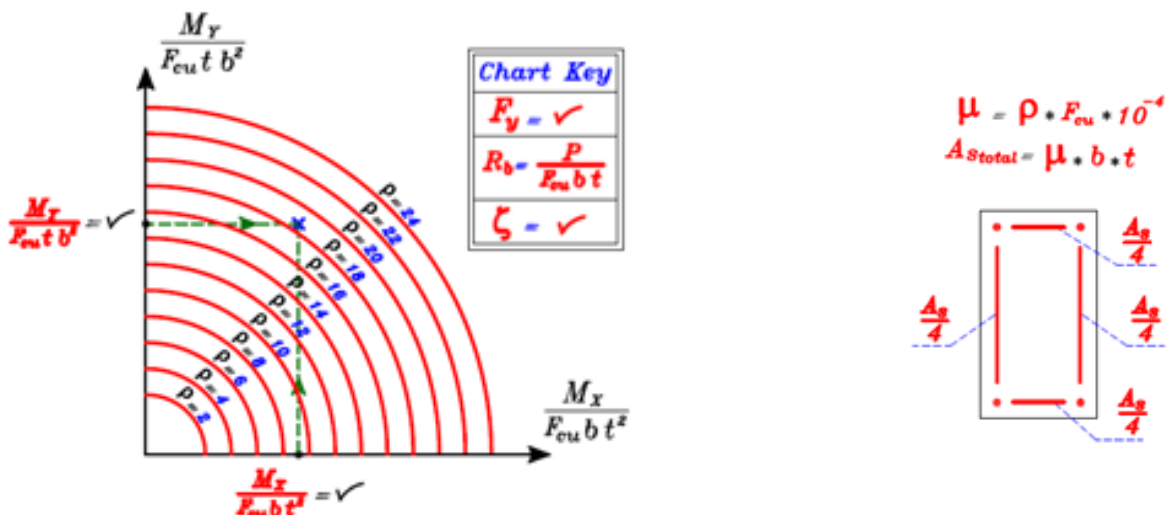
## 1- Biaxially Loaded Columns- Biaxial Interaction Diagram Method

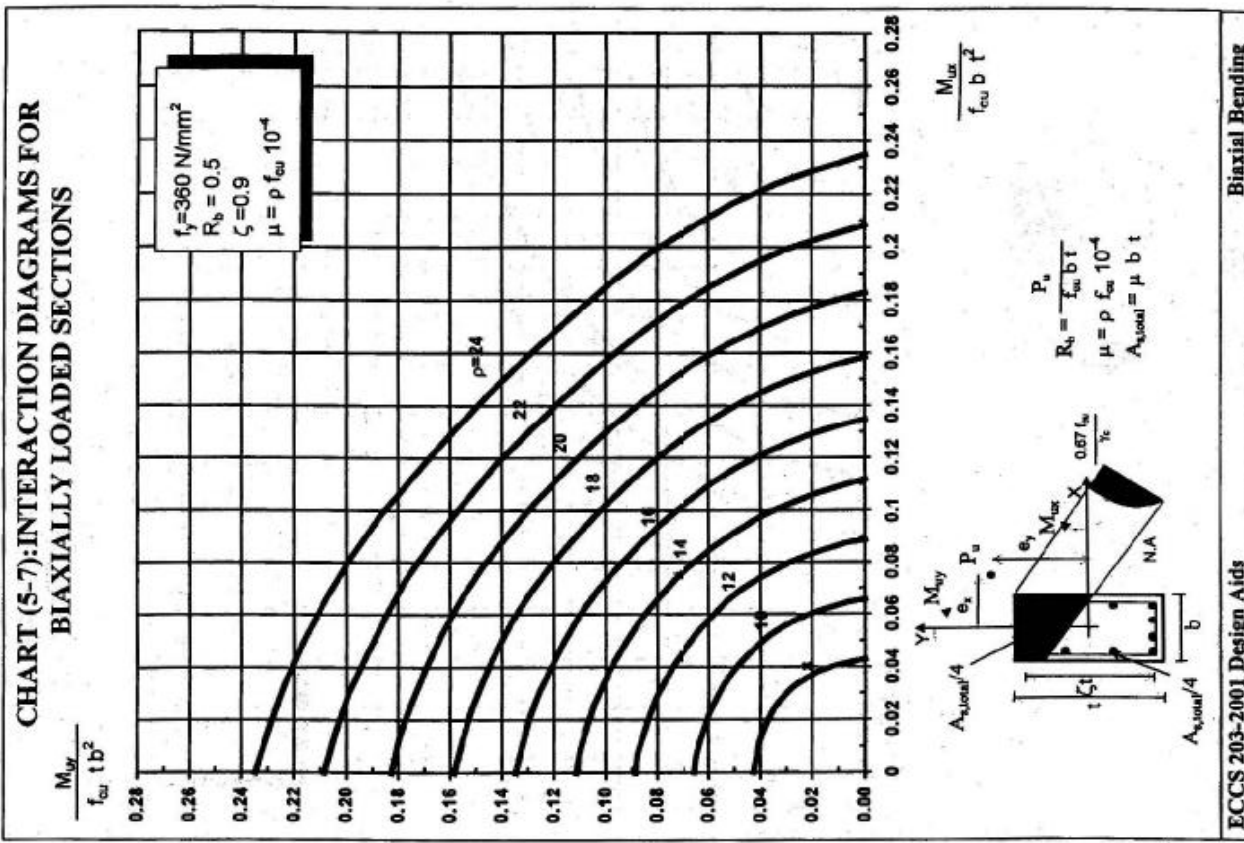
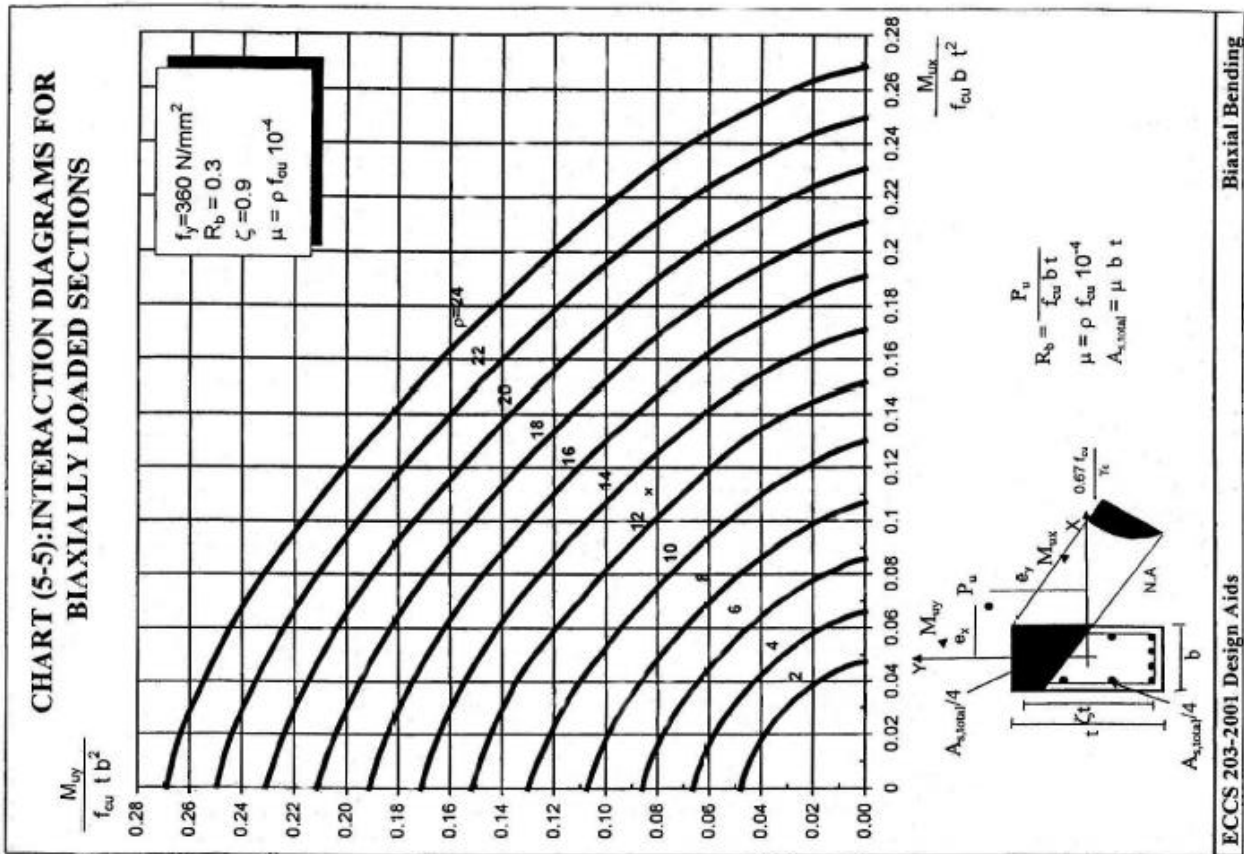
1- Using numerical integration, the force and moment in the concrete zone are determined by integrating the nonlinear idealized stress-strain curve for concrete in compression. 3-D failure surfaces are drawn for the column. The biaxial interaction diagrams (curves) are produced by cutting a horizontal plan  $R_b$  (load contour) through the failure surface as shown in the figure.



2- The use of these interaction curves is as follows:

- Determine the curve using the calculated of  $\{f_y, R_b, (M_y / f_{cu} t b^2), (M_x / f_{cu} b t^2), \xi\}$
- From the design ID, get  $\mu, \rho$  and calculate  $A_{s \text{ total}} = \mu * b * t$
- $A_{s \text{ total}} (\geq A_{s \text{ min}} = 0.006 b t)$  is distributed equally on the four sides.



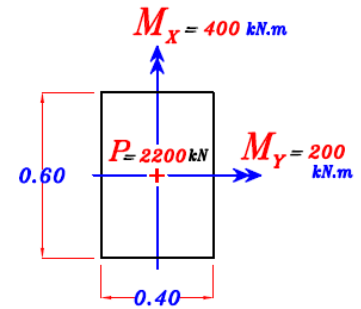


## 2- Biaxial Design Example for the Biaxial Interaction Diagrams

Design the shown rectangular section if

$f_{cu} = 25 \text{ MPa}$  &  $f_y = 360 \text{ MPa}$ ,  $P_u = 2200 \text{ kN}$ ,

$M_{xu} = 400 \text{ m.kN}$ ,  $M_y = 200 \text{ m.kN}$



assume  $\zeta = 0.90$  ----- ECCS لا توجد قيمه غيرها فى ال

$$R_b = \frac{P}{F_{cu} b t} = \frac{2200 \cdot 10^3}{25 \cdot 400 \cdot 600} = 0.366 \rightarrow \text{Not in ECCS}$$

لانه لا توجد قيمه ل  $R_b = 0.366$  فى كتاب ECCS فيتم حساب قيمتين ل  $\rho$  مره عند  $R_b = 0.30$  و مره عند  $R_b = 0.40$  ثم أخذ قيمه ل  $\rho$  بينهم

For  $R_b = 0.30 \rightarrow$  ECCS Page (5-13)

$$\left. \begin{aligned} \frac{M_x}{F_{cu} b t^2} &= \frac{400 \cdot 10^6}{25 \cdot 400 \cdot 600^2} = 0.111 \\ \frac{M_y}{F_{cu} t b^2} &= \frac{200 \cdot 10^6}{25 \cdot 600 \cdot 400^2} = 0.083 \end{aligned} \right\} \rho = 11.8$$

For  $R_b = 0.40 \rightarrow$  ECCS Page (5-14)

$$\left. \begin{aligned} \frac{M_x}{F_{cu} b t^2} &= \frac{400 \cdot 10^6}{25 \cdot 400 \cdot 600^2} = 0.111 \\ \frac{M_y}{F_{cu} t b^2} &= \frac{200 \cdot 10^6}{25 \cdot 600 \cdot 400^2} = 0.083 \end{aligned} \right\} \rho = 15$$

نأخذ  $\rho$  قيمه بينهم

To get value of  $\rho$  For  $R_b = 0.366$

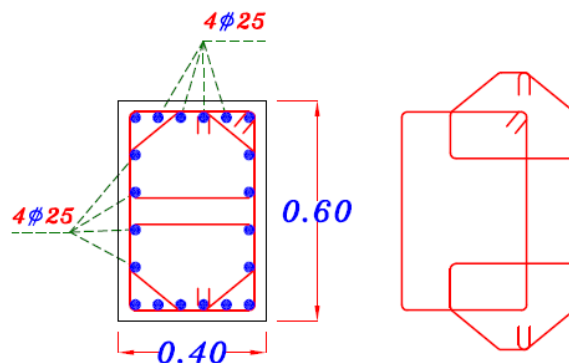
$$\left. \begin{aligned} R_b = 0.30 &\rightarrow \rho = 11.8 \\ R_b = 0.40 &\rightarrow \rho = 15 \end{aligned} \right\} \rho = 13.4 \text{ نأخذ قيمه بينهم}$$

$$\mu = \rho * F_{cu} * 10^{-4} = 13.4 * 25 * 10^{-4} = 0.0335$$

$$A_{S_{total}} = \mu * b * t = 0.0335 * 400 * 600 = 8040 \text{ mm}^2$$

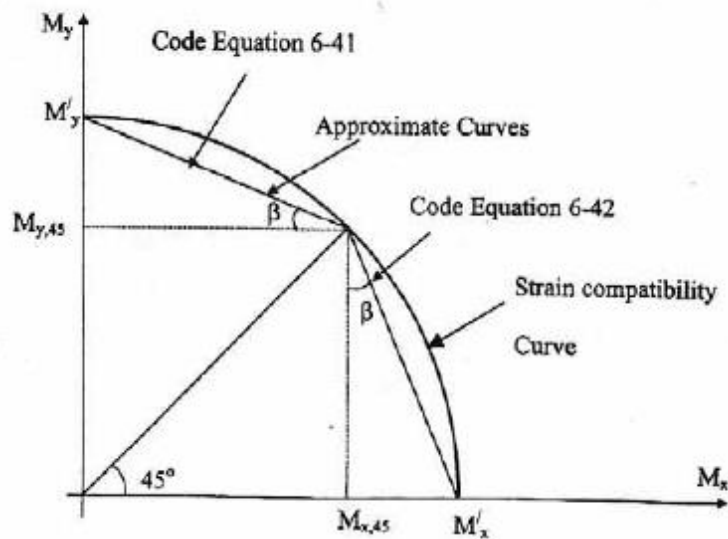
$$- \text{Check } A_{S_{min}} = \frac{0.6}{100} * b * t = \frac{0.6}{100} * 400 * 600 = 1440 \text{ mm}^2$$

$$A_s = A_{S_{total}} = 8040 \text{ mm}^2 \quad \textcircled{20 \phi 25}$$



### 3- Biaxially Loaded Columns- ECP Code Method

- 1- ECP offers a simplified method for the case of a R-section with symmetrical reinforcement by approximating the curved shape of biaxial I.D. by two straight lines as shown in the figure. The biaxial state is transferred to a case of uniaxial eccentric compression.



- 2- If the column section under  $P_u$ ,  $M_{ux}$  &  $M_{uy}$  not given, assume the section as follows:

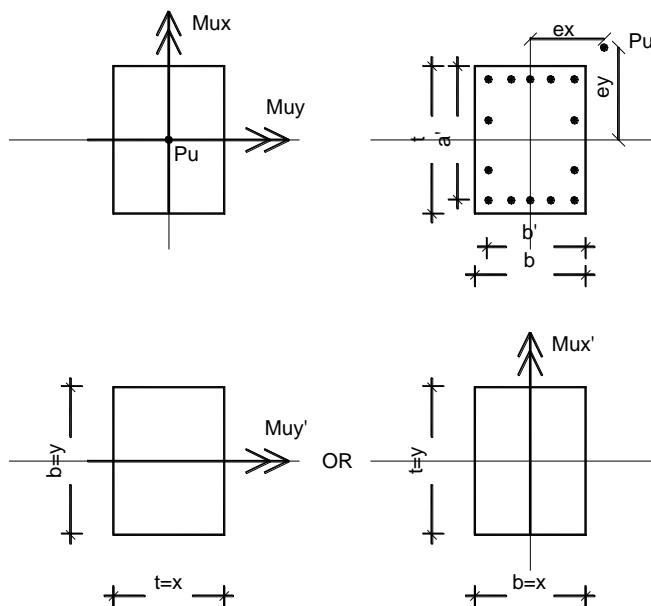
$$e_y = M_{ux} * 10^6 / (P_u * 10^3)$$

$$e_x = M_{uy} * 10^6 / (P_u * 10^3)$$

If  $e_x = e_y$  take  $x = y$  square sec.

If  $e_x > e_y$  take  $x > y$  R-sec.

If  $e_y > e_x$  take  $y > x$  R-sec.



Case ( 2 )

Case ( 1 )

3- In case of rectangular or square columns sections with equal steel on its faces (uniform steel arrangement), then design moments can be reduced into an equivalent uniaxial moment as given below. (Consider  $a' = y - 50 \text{ mm}$ ,  $b' = x - 50 \text{ mm}$  as shown in the figure)

4- **Case (1):** If  $M_{ux} / a' > M_{uy} / b'$

then design the section for  $P_u$  &  $M_{ux} = (M_{ux} + \beta (a'/b') M_{uy})$  using I.D. of uniaxial bending

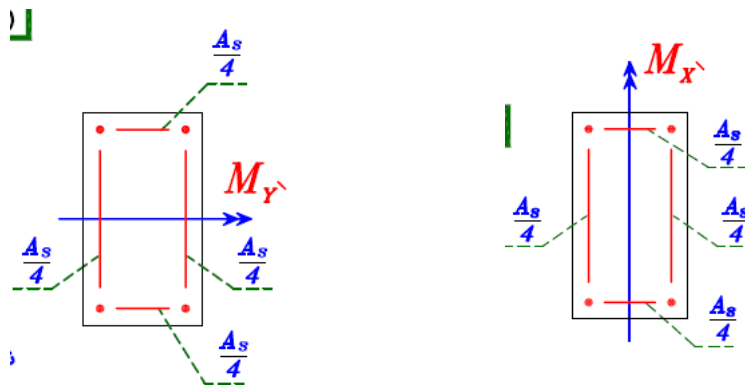
5- **Case (2):** If  $M_{ux} / a' \leq M_{uy} / b'$

then design the section for  $P_u$  &  $M_{uy} = (M_{uy} + \beta (b'/a') M_{ux})$  using I.D. of uniaxial bending.

**Table for values of the coefficient  $\beta$  :-**

$P_u * 10^3 / (f_{cu} bt)$	$\leq 0.2$	0.3	0.4	0.5	$\geq 0.6$
$\beta$	0.80	0.75	0.70	0.65	0.60

6- The calculated steel area from I.D.  $A_s \text{ total} \geq A_s \text{ min} (= 0.006 bt)$



#### **4- Biaxial Design Example ECP Code Method (Case 1)**

**Given:**  $f_{cu} = 25 \text{ MPa}$  ,  $f_y = 360 \text{ MPa}$  ,  $d' = 50 \text{ mm}$

$$P_{D.L.} = 1000 \text{ kN} \quad , \quad P_{L.L.} = 1500 \text{ kN}$$

$$M_{X.D.L.} = 400 \text{ kN.m} \quad , \quad M_{X.L.L.} = 500 \text{ kN.m}$$

$$M_{Y.D.L.} = 200 \text{ kN.m} \quad , \quad M_{Y.L.L.} = 300 \text{ kN.m}$$

**Required:** Design the column section

$$P_u = 1.4 P_{D.L.} + 1.6 P_{L.L.} = 3800 \text{ kN} \quad \text{where } P_{L.L.} > 0.75 P_{D.L.}$$

$$M_{ux} = 1.4 M_{X.D.L.} + 1.6 M_{X.L.L.} = 1360 \text{ kN.m} \quad \text{where } M_{X.L.L.} > 0.75 M_{X.D.L.}$$

$$M_{uy} = 1.4 M_{YDL} + 1.6 M_{YLL} = 760 \text{ kN.m where } M_{YLL} > 0.75 M_{YDL}$$

$$e_y = M_{ux} * 10^6 / (P_u * 10^3) = 357.9 \text{ mm}$$

$$e_x = M_{uy} * 10^6 / (P_u * 10^3) = 200.0 \text{ mm}$$

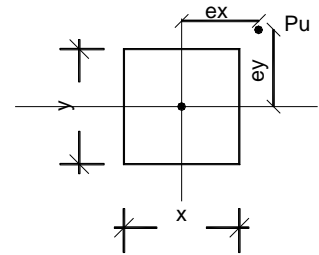
Assume square or rectangular section

If  $e_x = e_y$  take  $x = y$  square sec.

If  $e_x > e_y$  take  $x > y$  R-sec.

If  $e_y > e_x$  take  $y > x$  R-sec.

Assume  $x = 800 \text{ mm}$  ,  $y = 1000 \text{ mm}$  ( $e_y > e_x$ )



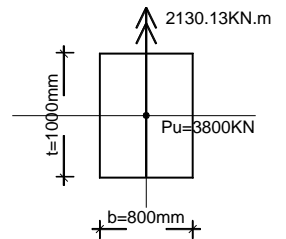
$$a' = t - d' = 1000 - 50 = 950 \text{ mm, } b' = b - d' = 800 - 50 = 750 \text{ mm}$$

$$M_{ux} / a' = 1.432, \quad M_{uy} / b' = 1.013$$

$$M_{ux} / a' > M_{uy} / b' \quad \text{then } M'_{ux} = M_{ux} + \beta (a' / b') M_{uy}$$

$$P_u * 10^3 / (f_{cu} b t) = 0.19, \quad \text{then } \beta = 0.8 \text{ from the previous table}$$

$$M'_{ux} = 1360 + 0.8 * (950 / 750) * 760 = 2130.13 \text{ kN.m}$$



Then design a column as shown in the Figure

$$e = M'_{ux} * 10^6 / (P_u * 10^3) = 560.6 \text{ mm}$$

$$e / t = 0.561 \quad (1)$$

$$K = P_u * 10^3 / (f_{cu} b t) = 3800 * 10^3 / (25 * 800 * 1000) = 0.19 \quad (2)$$

For  $f_y = 360 \text{ MPa}$  (uniform steel arrangement)

$$\text{and } \xi = (t - 2d') / t = (1000 - 2*50) / 1000 = 0.9 \quad \text{then use chart No. ....}$$

From the chart get  $\rho = 5.2$

$$\mu = \rho f_{cu} * 10^{-4} = 0.013 = 1.3 \%$$

$$A_s = \mu b t = 10400 \text{ mm}^2 \quad (\text{use } 24 \Phi 25)$$

Distribute the steel uniformly over the section

